Review on Digital Image Restoration Methods and Future Directions

K. Praveen Kumar, Dr. C. Venkata Narasimhulu, Dr. K. Satya Prasad

Abstract— The aim of image restoration is to estimate the original image from an observation image degraded by blur and additive noise as much as possible. Different image restoration methods have developed by many researchers. The methods reviewed in this paper are Non Linear Total Variation, PDE noise removal algorithm, Hybrid image restoration and Wavelet Analysis method. These methods having some drawbacks like stair case effect, blocky effect without gradient parameter, proper estimation of PSF, artifacts in the image respectively. The analysis and discussions indicate to that wavelet frame is a new way of solving PDEs in general, which will impart a new insight that will enrich the existing theory and applications of numerical PDEs, as well as, those of wavelet frames to get better results.

Index Terms— Image Restoration, PDE, Total Variation, Wavelet Analysis.

1 INTRODUCTION

THE image gets degrade in the process of image acquisition, transmission, storage and procession due to blur and noise. Finite aperture size results optical blur, finite sensor size causes sensor blur and includes motion blur due to finite aperture time. The original image is to estimate from the observed degr aded image. The restoration of degraded images can be applied in many application areas like remote sensing images, and medical imaging photos etc. Generally, the aim is to estimate unknown original image $x \in R^n$from the noisy image $y \in R^m$ that is often modeled as

$$y = Ax + \eta \quad (1)$$

Where A is some linear bound operator, typically a convolution operator in image deconvolution, and η is a white Gaussian noise in the observed image with variance σ^2.

Restoration of original image from the degraded image when the knowledge of degradation function unknown the problem becomes blind image restoration. With known degraded function the problem becomes non blind image restoration. The types of noises generally includes in digital images are Gaussian Noise, Salt and Pepper Noise, Rayleigh Noise, Gamma Noise, Uniform Noise and Impulse Noise. The main area of application is image reconstruction in radio astronomy, radar imaging and tomography.

2 IMAGE RESTORATION METHODS

2.1 Non linear total variation based Restoration

This paper [1] discussed constrained noise minimizing problem that is given by equation (2)

$$\text{minimize} \int_{\Omega} \sqrt{u_x^2 + u_y^2}\, dx\, dy \quad (2)$$

subject to constraints involving u_o.

Here they wish to reconstruct $u(x,y)$ from degraded image $u_o(x,y)$ for $x, y \in \Omega$. The two constraints taken in this algorithm are the white Gaussian noise $n(x,y)$ is of zero mean.

$$\int_{\Omega} u\, dx\, dy = \int_{\Omega} u_o dx\, dy \quad (3)$$

And standard deviation

$$\int \frac{1}{2}(u - u_o)^2\, dxdy = \sigma^2 \quad (4)$$

Due to one is linear and one is non linear constraint, the minimisation problem arrived at Euler-lagrange equation.

$$0 = \frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right) + \frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right) - \lambda 1 - \lambda 2\,(u - u_o) \text{ in } \Omega, \text{ with} \quad (5)$$

$$\frac{\partial u}{\partial n} = 0 \text{ on the boundary of } \Omega = \partial\Omega. \quad (6)$$

The first constraint was dropped due to evolution procedure of the given problem. At the end the researchers concluded that this algorithm produces much clear discontinuities than wiener filter restoration method. The use more [2] constraints needed to remove distortion and to obtain nontrivial steady state. The experiments done in this paper indicates that the use of more constraints will give more details of the solution.

The figure 1 shows the experiment results done by the authors. The figures indicate that Total Variation method produces good results than the basic Wiener Filter restoration method.

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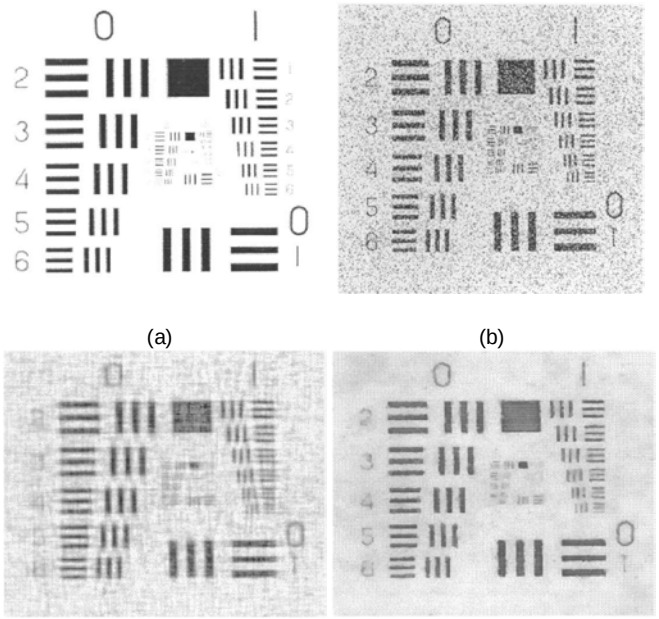

(a) (b)

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Fig. 1 Total variation based Restoration method Results [1] (a). Original Image (b) Noisy Image SNR=1.0 (c) Restored image using Wiener Filter from b (d) Restored image using TV from b.

2.2 Image Restoration using Wavelet Analysis

In this paper [3] the authors given the disadvantages of Wiener Filter, Regularized Filter, Lucy-Richardson algorithm, Blind Deconvolution algorithm and proposed the approach based on Wavelet analysis. They found that, it is required to pre process the degraded image before the restoration and analyze the degraded image to get more information [3]. In this the authors implemented two types of pre processes. Filtering in frequency domain before applying wavelet analysis and gray leve transformation based on histogram values. The high frequency components generally dominated by the noise, where edges of the image are available. Thus low pass filter are used to remove the noise.

Gray level transformation [4], [5] increase the contrast by extending the gray region to deblur the image. The selecting of appropriate parameter to increase contrast is very important. Wavelet analysis reveals important details of the image such as discontinuities and break points. Wavelet analysis is capable to perform local analysis and to analyze a signal without noticeable degradation. Single approach may not produce good results. From the above analysis they found that combined approaches with wavelet analysis is effective analysis to restore the image.

The results obtained from the experiments by the authors are given in figure 2. The selection of the wavelet function is also important and they adopted Coiflet wavelets

2.3 Fourth order PDE Noise Removal Method

Fourth order [6] PDE method can remove the noise during fourth-order PDE diffusion process without blocky. Most of the second order PDE methods like TV models and their variants having general problem that is image restored with blocky effect, since the blocky effect is internal characteristic second order equations. Second order PDE methods are also losing fine structures of image.

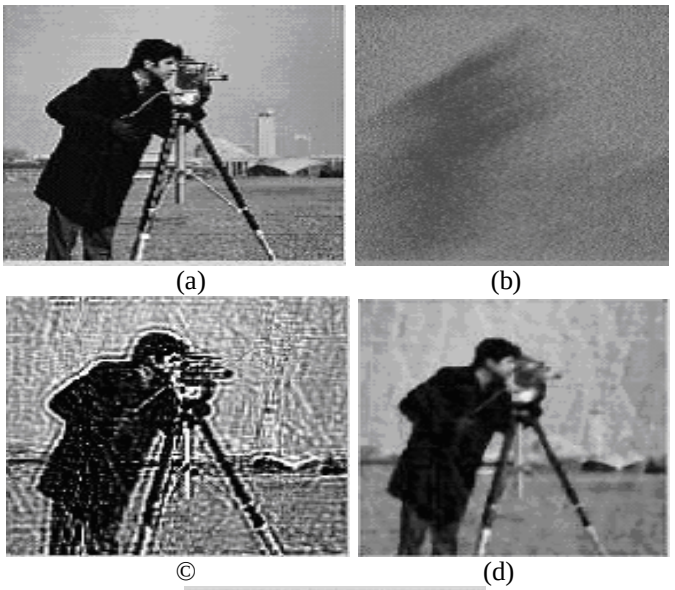

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Fig. 2. Results of [3] (a) Original Image (b) Degraded Image (c) Restored image using Wiener Filter directly (d) Restored image using preprocess and wavelet analysis (e) Restored image by using combined methods.

The formulation of fourth order PDE is given by

$$E(u)=\int_{\Omega} f(|\nabla^2 u|)\, d\Omega \qquad (7)$$

where E(u) is the energy factor, $\nabla^2 u$ is laplacian operator.

Laplacian operator produce a piecewise planar image other than level planer image when the function $f(.) \geq 0$ and is an increasing function $f^1(.)>0$. Minimization of the functional is nothing but smoothing the image since the functional is an increasing function with respect to smoothness of the image as measured by $|\nabla^2 u|$.

This analysis produced Euler equation

$$\nabla^2(C(|\nabla^2 u|)\nabla^2 u)=0 \qquad (8)$$

This can be solved using gradient descent procedure

$$\frac{\partial u}{\partial t} = \nabla^2(C(|\nabla^2 u|)\nabla^2 u) \qquad (9)$$

Where $$C(x)=\frac{f^1(|\nabla^2 u|)}{\nabla^2 u} \qquad (10)$$

The researchers found that [7,8] the bigger the diffusion magnitude is, the larger dissipation occurs. From the above

analysis fourth order PDE co-efficient is produced by laplacian operator of image only given in equation (11).

$$C(|\nabla^2 u|) \quad (11)$$

Another parameter introduced in the co-efficient to suppress the dissipation effect. The new co-efficient becomes

$$C(|\nabla^2 u| + |\nabla u|) \quad (12)$$

Now the diffusion equation of fourth order PDE becomes

$$\frac{\partial u}{\partial t} = \nabla^2(C(|\nabla^2 u| + |\nabla u|)\nabla^2 u) \quad (13)$$

Since the diffusion co-efficient is now sum of two aspects.

Gradient operator used for edge information. The researchers think that the (13) equation can protect the edge with good results during smooth operation.

The results obtained from the experiments by the authors are given in figure 3. The results indicating that fourth order PDE reducing the dissipation effect by revising the co-efficient by including gradient information. The images are processed at *t*=100 and *t*=200.

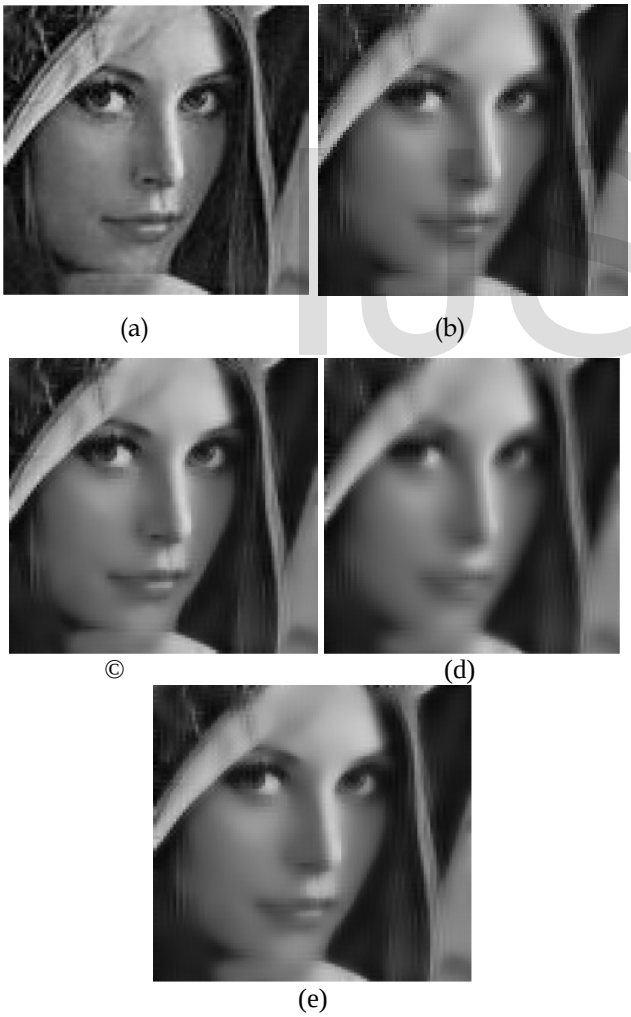

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Fig. 3. Fourth order PDE Noise Removal Method Results [6] (a) Original Image (b) Original Method when t=100 (b) Improved Method when t=100 (c) Original Method when t=200 (d) Improved Method when t=200.

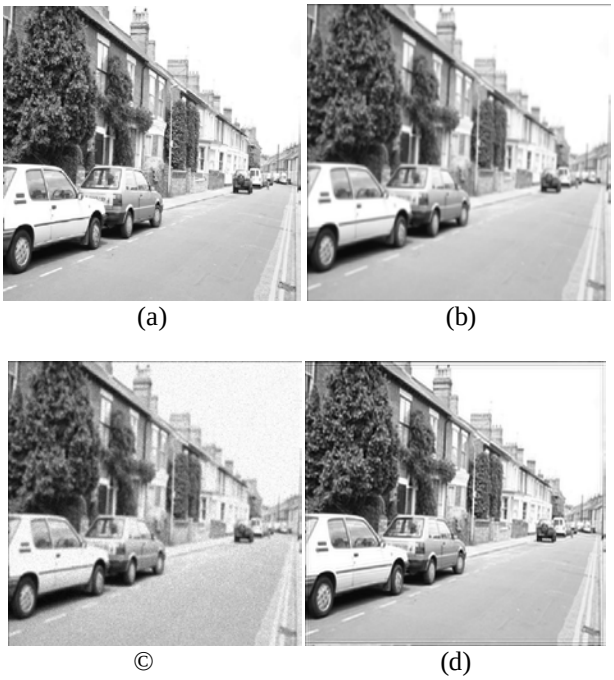

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Fig. 4. Results of [9] (a) Original Image (b) Degraded Image with blurring (c) Blurred image with noise (d) Restored image by proposed technique

2.4 Hybrid Image Restoration Method

Hybrid image restoration method [9] having Stationary Wavelet Transform (SWT) denoising method based on smoothing of coefficient in wavelet domain and deblurring based on regularisation in frequency domain using image prior.

Inherent characteristics of redundancy of SWT co-efficients, the noise can be recognised and removed. SWT method decompose the image in to sub bands and co-effcients of these sub bands are smoothed for denoising. The next step of denoising is dublurring, it done by regularization based on available prior information to make edge sharpener.

In SWT the image is divided into sub bands using orthogonal wavelet transform at level j. The selected threshold value T eliminate small magnitude wavelet co-efficient of equation (14) and impact of [10] large co-efficient are decreased.

$$W_\psi^i(j,m,n) = \frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\psi_{j,m,n}^i(x,y) \quad (14)$$

where ψ_j^i(j,m,n) is the wavelet function.

The denoised co-efficient

$$\hat{W}_\psi^i(j,m,n) = \begin{cases} W_\psi^i(j,m,n) - T & W_\psi^i(j,m,n) \geq T \\ W_\psi^i(j,m,n) + T & W_\psi^i(j,m,n) \leq T \\ 0 & otherwise \end{cases} \quad (15)$$

The denoised image is generated by taking inverse wavelet transform of $\hat{W}_\psi^i$(j,m,n).

After SWT, assuming the image is still have the noise and not estimated accurately using Point spread Function (PSF), the researchers used the prior information about the image within MAP (Maximum a Posteriori) framework. For given g, MAP explanation for f is given by equation (16).

$$f=\arg\max_f P(f/g)\propto P(g/f)P(x) \quad (16)$$

Where $$P(x)=e^{\alpha\sum_{i,k}\rho(b_{i,k}*f)} \quad (17)$$

$$P(g/f)\propto e^{\frac{1}{2n^2}\|f-C_f g\|^2} \quad (18)$$

c_f is a convolution matrix of size NxN, η is variance of noise, i is sums over image pixels, $b_{i,k}$ is the k^{th} filter centered at pixel i, ρ is the heavily tailed function.

For minimization of MAP explanation for f, they taken log of (16),(17) and (18) equations hence the process becomes minimization of cost function equation given by (19)

$$\|g-C_f f\|^2+w\sum_{i,k}\rho(b_{i,k}*f) \quad (19)$$

Where $w=\alpha\eta^2$.

To detect edges in the image edge detector b_k is developed by the researchers to apply prior information.

The results obtained from the experiments by the researchers are given in figure 4. From the results the use of SWT for denoises the image and PSF estimation makes the deconvolution simpler. And the introduction of an image prior produces better results in frequency domain.

This review can be summarized as given by table I

TABLE I.

TABULATION FOR EXISTED REVIEWED RESTORATION METHODS

Method	Advantages	Disadvantages
Non Linear Total Variation Restoration Method [1]	Discontinuities much clear than wiener filter method	Creates the staircase effect for images
Restoration using Wavelet Analysis [3]	Combined approach produce good results than single approach	Produces artifacts
Fourth order PDE Noise Removal Method [6]	Remove blocky effect due more dissipation and improve visual effects	Without gradient parameter higher order PDE produces more dissipation.
Hybrid Image Restoration Method [9]	SWT with PSF estimation produces better ISNR	Noise will still exist if Initial estimation of PSF is not well

3 CONCLUSION AND FUTURE WORK

In this paper we have presented review on different restoration methods for restoration of original image from the degraded image. The proposed total variation method is especially effective on restoring images that are piece wise constant. Since the Wavelet frame based systems are redundant systems, the mapping of the image and its coefficients is not one to one basis and representation of image is not unique. Hybrid image method having blind image restoration method is effective if initial point spread function must be known. If the researchers can establish connection between wavelet frame base method and various PDE based methods that include the total variation model, non linear diffusion PDE based methods; they can develop new technique to restore the image with better results.

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